1) $A[1,0,1],\ B[3,2,1],\ C[0,0,4],\ D[5,4,z]$

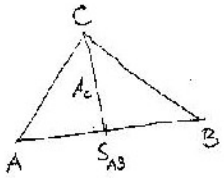
a) ρ_{ABC}: $\vec{u} = \vec{AB} = B-A = (2;2;0)$
$\vec{v} = \vec{AC} = C-A = (-1,0,3)$

obecná rce $\Rightarrow$ potřebuji normálový vektor $\vec{n} = \vec{u}\times\vec{v} = \begin{vmatrix} 2 & 0 & 2 & 2 \\ 0 & 3 & -1 & 0 \end{vmatrix} = (6;-6;2) \sim (3,-3,1)$

$$\left.\begin{aligned} \rho:&\ 3x-3y+z+D=0 \\ A\in\rho:&\ 3-0+1+D=0 \\ &D=-4 \end{aligned}\right\}\ \underline{\underline{\rho:\ 3x-3y+z-4=0}}$$

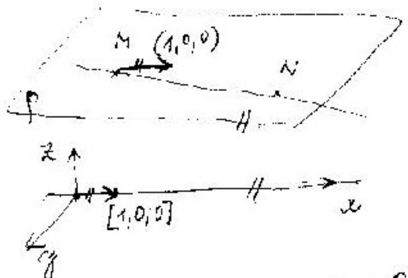
b) $D\in\rho$: $3\cdot5 - 3\cdot4 + z - 4 = 0$
$15 - 12 - 4 = -z$
$\underline{\underline{z = 1}}$

c)



$$\left.\begin{aligned} S_{AB} &= \frac{A+B}{2} = [2;1;1] \\ \vec{u} &= \vec{CS}_{AB} = (2;1,-3) \\ C &\in A_c \end{aligned}\right\}\ \underline{\underline{A_c:\ \begin{aligned} x &= 2t \\ y &= t \\ z &= 4-3t,\ t\in\langle 0;1\rangle \end{aligned}}}$$

2)



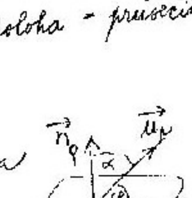
$$\left.\begin{aligned} \rho \parallel x &\Rightarrow \vec{u} = \vec{u}_x = (1,0;0) \\ M,N\in\rho &\Rightarrow \vec{v} = \vec{MN} = (4;4;5) \end{aligned}\right\}\ \vec{n} = \vec{u}\times\vec{v} = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 4 & 5 & 4 & 4 \end{vmatrix} = (0,-5,4)$$

$$\left.\begin{aligned} \rho:&\ -5y+4z+D=0 \\ M\in\rho:&\ -5(-2)+4(-4)+D=0 \\ &10-16+D=0 \\ &D=6 \end{aligned}\right\}\ \begin{aligned} &\cancel{\rho: 3y+4z+10=0} \\ &\rho:\ +5y-4z-6=0 \\ &\underline{\underline{-5y+4z+6=0}} \end{aligned}$$

3) p: $x = 1+t$
$y = 2-t$
$z = t;\ t\in R$ $\quad\rho$: $3y+8=0$

a) vzáj. poloha - průsečíky $\Rightarrow$ $3\cdot(2-t)+8=0$
$6-3t+8=0$
$3t=14$
$t=\frac{14}{3}$ $\rightarrow$ 1 řešení = 1 průsečík $P\left[\frac{17}{3};-\frac{8}{3};\frac{14}{3}\right]$

b) odchylka



$\vec{n}_\rho = (0;3;0)$
$\vec{u}_p = (1,-1,1)$
$\cos\alpha = \frac{\vec{n}_\rho\cdot\vec{u}_p}{|\vec{n}_\rho|\cdot|\vec{u}_p|} = \frac{|0-3+0|}{3\cdot\sqrt{3}} = \frac{1}{\sqrt{3}} \rightarrow \alpha \doteq 54°45'$
$\varphi = 90° - 54°45' = 35°15'$

4) $x^2+y^2-6x-10y+9=0$

$(x^2-6x+9)+(y^2-10y+25)=9+25-9$

$(x-3)^2+(y-5)^2=25$

a) $\underline{S[3;5]\,;\ r=5}$

b) průsečíky p osou x   osa x: $x=t$, $y=0$, $z=0$; $t\in R$

$t^2+0^2-6t+0+9=0$

$t^2-6t+9=0$

$(t-3)^2=0$

$t_{1,2}=3 \Rightarrow$ 1 řešení = 1 průsečík $\rightarrow$ přímka je tečna

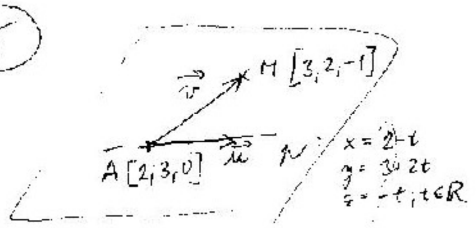
c) $T[-1;2]$ t: $x_0 x+y_0 y-3x_0-3x-5y_0-5y+9=0$

$-x+2y+3-3x-10-5y+9=0$

$-4x-3y+2=0$

$\underline{4x+3y-2=0}$

(5)



$\vec{u}=\vec{u_p}=(-1;2;-1)$

$\vec{v}=\vec{AM}=(1;-1;-1)$

ρ: $x=2-t+s$

$y=3+2t-s$

$z=0-t-s$; $t,s\in R$

A

1

1) $9x^2 + 16y^2 - 54x + 64y + 1 = 0$

$9(x^2 - 6x + 9) + 16(y^2 + 4y + 4) = -1 + 81 + 64$

$9(x-3)^2 + 16(y+2)^2 = 144$

$\frac{(x-3)^2}{16} + \frac{(y+2)^2}{9} = 1$

elipsa – 1b

$S[3; -2]$ – 1b

$a = 4$, $b = 3$ – 1b

$e = \sqrt{16-9} = \sqrt{7}$ – 1b

2) $y + 2 = 0$

$x^2 + 6x - y + 7 = 0$

$x^2 + 6x + 2 + 7 = 0$

$x^2 + 6x + 9 = 0$

$D = 36 - 36 = 0 \Rightarrow$ priamka je dotyčnica

$(x+3)^2 = 0$

$x = -3;\ y = -2$ $[-3; -2]$

3) $x^2 + y^2 + 4x - 8y + 10 = 0$

$3x - y + c = 0 \Rightarrow y = 3x + c$

$x^2 + 9x^2 + 6xc + c^2 + 4x - 24x - 8c + 10 = 0$

$10x^2 + (6c - 20)x + c^2 - 8c + 10 = 0$ 1b

$D = 36c^2 - 240c + \cancel{400} - 40c^2 + 320c - \cancel{400} = 0$

$-4c^2 + 80c - \cancel{400} = 0 \quad /:(-4)$

$c^2 - 20c + \cancel{100} = 0$

$\cancel{c_{1,2} = \frac{20 \pm \sqrt{400-400}}{2}}$

$c_1 = 0$

$c_2 = 20$ 2b

$\cancel{10x^2 + 40x + 30 = 0}$

$\cancel{x^2 + 4x + 3 = 0}$

$\cancel{(x+1)(x+3) = 0}$

$\cancel{x_1 = -1}$ $\cancel{x_2 = -3}$

$\cancel{y_1 = 4}$ $\cancel{y_2 = 1}$

$c_1 = 0$: $x = \frac{20 - 6c}{20} = 1 - 0{,}3c = 1$

$y_1 = 3$ $T[1, 3]$

2b

$c_2 = 20$: $x_2 = 1 - 6 = -5$ $T[-5; 5]$

$y = 3(-5) + 20 = 5$

4) $S[1; -1]$ $a = 2$, $b = 1$

$\frac{(x-1)^2}{4} + \frac{(y+1)^2}{1} = 1$

$x^2 - 2x + 1 + 4y^2 + 8y + 4 - 4 = 0$

$x^2 - 2x + 1 + 4y^2 + 8y + 1 = 0 \Rightarrow$ c

$x^2 + 4y^2 - 2x + 8y + 1 = 0$

5) $(x-3)^2 - 4(y+2)^2 = 11 + 9 - 16$

$\frac{(x-3)^2}{4} - \frac{(y+2)^2}{1} = 1$

$M[5; y_0]$: $25 - 4y_0^2 - 30 - 16y_0 - 11 = 0$

$4y_0^2 + 16y_0 + 16 = 0$

$y_0^2 + 4y_0 + 4 = 0$

$y_{1,2} = \frac{-4 \pm \sqrt{16-16}}{2} = -2$

$M[5; -2]$

t: $\frac{2(x-3)}{4} - 0 = 1$

$2x - 6 = 4$

$2x - 10 = 0$

$x - 5 = 0$